Babasaheb Bhimrao Ambedkar Bihar University, Muzaffarpur
Directorate of Distance Education
T.D.C. 3rd Semester Examination 2015 (Session 2014-17)
Subject:- Mathematics (Hons.)
Paper – 3rd
Model Paper (Full Marks – 80)

1(a) Define Convergence of a sequence and prove that the Limit of a sequence is unique.

(b) Prove that the sequence $\langle a_n \rangle$ defined by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2a_n}$ Converges to 2.

2(a) State and prove Higher Logarithmic Test for the convergence for a positive term infinite Series.

(b) Discuss the Convergence of the series whose nth term is given by:

$$u_n = \frac{(a+nx)^n}{\lfloor n}$$

3(a) Define absolute Convergence of an alternating Series and prove that every absolutely Convergent is Convergent but the Converse is not necessarily true.

(b) Discuss the Convergence of the series

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots \text{ to } \infty.$$

4(a) Prove that the set of all nth roots of unity froms a multiplicative Group.

(b) Every group of prime order is cyclic. Prove this.

5(a) Prove that the union of two Subgroups may not be a Subgroup.

(b) If a, b are any two elements of a group G and H any Subgroup of G then prove that

$$H_a = H_b \Rightarrow ab^{-1} \in H \;\&\; H \;\&\; aH = bH \Rightarrow a^{-1}b \in H$$